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Scattering Matrices of Junction Circulator with Chebyshev Characteristics

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***Abstract*—The purpose of this paper is to derive the scattering matrix of junction circulators with Chebyshev characteristics. This is done by forming the overall eigenvalues of the circulator one at a time in terms of the *ABCD* matrix of the matching network and the initial set of the junction eigenvalues. This paper deals both with the case where the frequency variation of the in-phase eigennetwork at the gyrator terminals is neglected compared to that of the counterrotating ones, and with the case where it is included. It is found that the former approach is in excellent agreement with the results obtained by assuming a 1-port model for the circulator. The influence of this eigennetwork on the overall frequency response is studied separately by combining the electromagnetic and network problems in the case of the stripline circulator.**

INTRODUCTION

THE THEORY of wide-band circulators using external matching networks usually starts by assuming that the equivalent circuit of the device is a 1-port network [1]–[8]. This 1-port circuit consists of a shunt conductance in parallel with either a lumped or distributed resonator. It assumes that the frequency behavior of the in-phase eigennetwork at the gyrator terminals may be omitted compared with that of the two counterrotating ones. The bandwidth over which this approximation applies has been discussed in [8] in terms of the resonant frequencies of the counterrotating eigennetworks, but a fuller investigation of the omission of the frequency variation of the in-phase eigennetwork on the quality of this equivalent circuit appears desirable.

The most general representation of the 3-port circulator is in terms of the eigenvalues of the scattering matrix [9]. The eigenvalues of this matrix are reflection coefficients associated with the different ways of exciting the junction. The entries of the scattering matrix are constructed by taking linear combinations of these eigenvalues. This method therefore yields not only the reflection coefficient at the input port but also the transmission coefficients of the junction. The approach is quite general and applies to the m-port junction also. It starts by representing the matching network at each port by its *ABCD* matrix. The eigenvalues at the input terminals of the junction are then obtained one at a time in terms of the *ABCD* parameters and the initial set of eigenvalues at the gyrator terminals.

In this paper the boundary condition for circulators with Chebyshev frequency characteristics is first established at the terminals of the matching network in terms of the eigenvalues of the scattering matrix by omitting the frequency variation of the in-phase eigennetwork at the gyrator terminals and subsequently reintroducing it to study its influence on the overall frequency response. It is found that the former results are in excellent agreement with those obtained by connecting the matching network directly to the 1-port circuit [8].

The influence of the in-phase eigenvalue on the overall frequency response of the circulator is studied separately in the case of the stripline circulator by combining the

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electromagnetic and network problems. This result indicates that the in-phase eigennetwork cannot be neglected for high-quality communication circulators. However, the 1-port results can be used, provided the in-phase eigennetwork is tuned by an additional independent variable such as a thin metal post at the center of the junction or thin metal posts at the input terminals of the quarter-wave transformers.

EIGENVALUES OF THE SCATTERING MATRIX

The entries of the scattering matrix of junctions are usually directly constructed in terms of their eigenvalues. In the case of the 3-port junction depicted in Fig. 1, in terms of ideal 2-port gyrators of characteristic admittance Y_0 [10], the following standard equations apply [9]:

$$S_{11} = \frac{s_0 + s_{+1} + s_{-1}}{3} \quad (1)$$

$$S_{12} = \frac{s_0 + s_{+1} \exp(j2\pi/3) + s_{-1} \exp(-j2\pi/3)}{3} \quad (2)$$

$$S_{13} = \frac{s_0 + s_{+1} \exp(-j2\pi/3) + s_{-1} \exp(j2\pi/3)}{3}. \quad (3)$$

The eigenvalues of the scattering matrix are the reflection coefficients associated with each possible way of exciting the junction

$$s_0 = \exp(-j2\Theta_0) \quad (4)$$

$$s_{+1} = \exp[-j2(\Theta_1 + \Theta_{+1} + \pi/2)] \quad (5)$$

$$s_{-1} = \exp[-j2(\Theta_1 + \Theta_{-1} + \pi/2)]. \quad (6)$$

The angles of the eigenvalues are defined below. The normalized admittance eigenvalues are related to those of the scattering matrix by

$$y_0 = \frac{1 - s_0}{1 + s_0} = j \tan \Theta_0 \quad (7)$$

$$y_{+1} = \frac{1 - s_{+1}}{1 + s_{+1}} = j \tan(\Theta_1 + \Theta_{+1} + \pi/2) \quad (8)$$

$$y_{-1} = \frac{1 - s_{-1}}{1 + s_{-1}} = j \tan(\Theta_1 + \Theta_{-1} + \pi/2). \quad (9)$$

The equivalent circuits of the admittance eigenvalues y_{+1} and y_{-1} are short-circuited transmission lines of electrical length $\Theta_1 + \Theta_{\pm 1}$ as shown in Fig. 2. For a 3-port circulator for which $S_{12} = -1$, the equivalent circuit for the admittance eigenvalue y_0 is a quarter-wave-long open-circuited transmission line of length Θ_0.

The boundary conditions given by (7)–(9) are in terms of transmission lines with the same characteristic impedance as the input lines. One suitable approximation in terms of the magnetic and frequency variables based on uniform transmission lines is

$$y_0 = jy_0' \tan \Theta_0 \quad (10)$$

$$y_{+1} = -jy_1 \cot \Theta_1 + jy_{+1}' \tan \Theta_{+1} \quad (11)$$

$$y_{-1} = -jy_1 \cot \Theta_1 + jy_{-1}' \tan \Theta_{-1} = -jy_1 \cot \Theta_1 - jy_{+1}' \tan \Theta_{+1}. \quad (12)$$

The form of these equations is obtained by forming the input admittance of the networks, which will be described, in terms of *ABCD* matrices.

The equivalent circuit for the admittance y_{+1} is a transmission line of admittance y_{+1}' and electrical length Θ_{+1} in cascade with a short-circuited transmission line of admittance y_1 of electrical length Θ_1. A similar statement applies to the admittance y_{-1}. The equivalent circuit for the admittance eigenvalue y_0 remains unchanged since it is unaffected by the applied direct magnetic field used to bias the device. It is observed in passing that the angles in (10)–(12) are related to those in (7)–(9) through appropriate transformations.

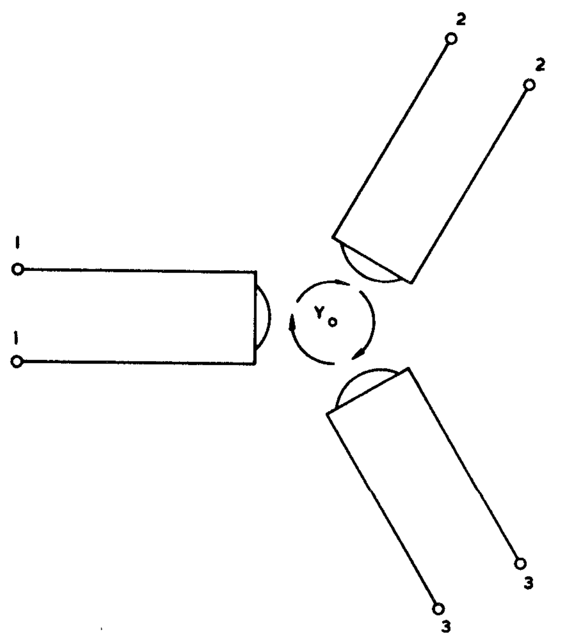

Fig. 1. Schematic of ideal 3-port junction circulator in terms of 2-port gyrators.

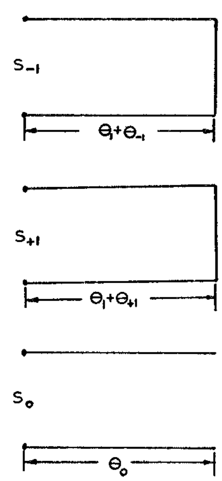

Fig. 2. Eigennetworks of 3-port junction circulator.

EIGENVALUES OF AUGMENTED SCATTERING MATRIX

If matching networks are now connected to each of the ports of the junction, one obtains the schematic diagram in Fig. 3. The eigennetworks for this circuit are shown in Fig. 4. The entries of the scattering matrix at the new terminals are now given by

$$\Gamma_{11} = \frac{\gamma_0 + \gamma_{+1} + \gamma_{-1}}{3} \tag{13}$$

$$\Gamma_{12} = \frac{\gamma_0 + \gamma_{+1} \exp (j2\pi/3) + \gamma_{-1} \exp (-j2\pi/3)}{3} \tag{14}$$

$$\Gamma_{13} = \frac{\gamma_0 + \gamma_{+1} \exp (-j2\pi/3) + \gamma_{-1} \exp (j2\pi/3)}{3}. \tag{15}$$

The eigenvalues at the input terminals of the $ABCD$ networks are given by straightforward calculation

$$\gamma_0 = \exp (- j2\psi_0) \tag{16}$$

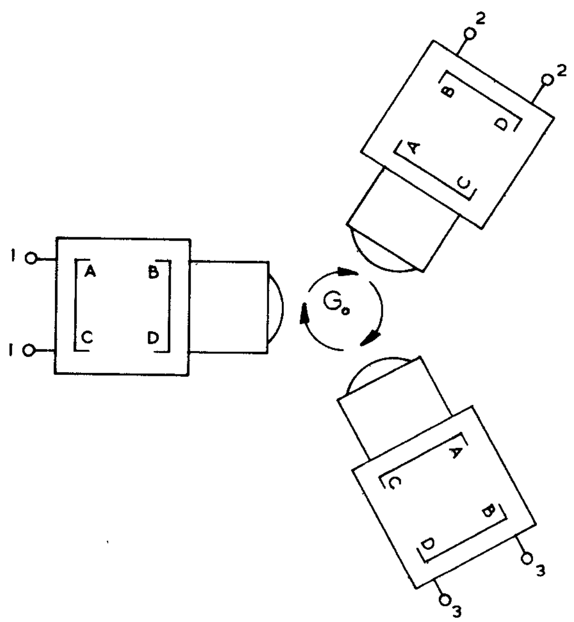

Fig. 3. Schematic of 3-port junction circulator with matching networks.

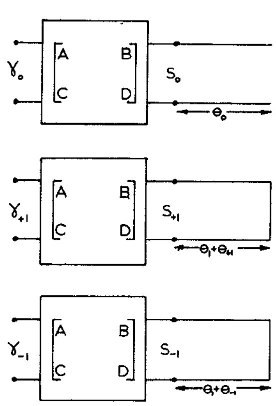

Fig. 4. Eigennetworks of 3-port circulator with matching networks.

$$\gamma_{+1} = \exp [-j2(\psi_{+1} + \pi/2)] \tag{17}$$

$$\gamma_{-1} = \exp [-j2(\psi_{-1} + \pi/2)] \tag{18}$$

where

$$j \tan (\psi_0) = \frac{jC + Dy_0}{A + jBy_0} \tag{19}$$

$$j \tan (\psi_{+1} + \pi/2) = \frac{jC + Dy_{+1}}{A + jBy_{+1}} \tag{20}$$

$$j \tan (\psi_{-1} + \pi/2) = \frac{jC + Dy_{-1}}{A + jBy_{-1}}. \tag{21}$$

It is also observed that each eigenvalue satisfies $\gamma\gamma^* = 1$. Hence, the new eigenvalues lie on the unit circle also.

For a single quarter-wave transformer, the $ABCD$ parameters are

$$A = \cos \Theta \tag{22}$$

$$B = \frac{\sin \Theta}{y_t} \tag{23}$$

$$C = y_t \sin \Theta \tag{24}$$

$$D = \cos \Theta. \tag{25}$$

The frequency variable Θ is

$$\Theta = \tfrac{1}{2}\pi(1 + \delta) \tag{26}$$

where

$$2\delta = 2\left(\frac{\omega - \omega_0}{\omega_0}\right). \tag{27}$$

At the band edges the frequency variable becomes

$$2\delta_0 = 2\left(\frac{\omega_1 - \omega_0}{\omega_0}\right). \tag{28}$$

The Chebyshev polynomial passes through zero for $n = 2$ when [8]

$$(2)^{1/2} \cos \Theta = \cos \Theta_0. \tag{29}$$

Fig. 5 depicts the frequency response considered in this text.

CIRCULATION ADJUSTMENT

The design proceeds by having the frequency at which the reflection coefficient passes through its zeros and maxima coincide with those of a Chebyshev polynomial.

When the reflection coefficient passes through zero, the eigenvalues lie equally spaced on the unit circle

Fig. 5. Chebyshev response of $n = 2$ quarter-wave coupled circulator.

$$\psi_{+1} + \pi/2 = \psi_0 + \pi/3 \quad (30)$$

$$\psi_{-1} + \pi/2 = \psi_0 - \pi/3. \quad (31)$$

Substituting the last two equations into (19)–(21) gives

$$j \tan \psi_0 = -j(D/B) \quad (32)$$

$$j \tan (\psi_0 + \pi/3) = \frac{jC + Dy_{+1}}{A + jBy_{+1}} \quad (33)$$

$$j \tan (\psi_0 - \pi/3) = \frac{jC + Dy_{-1}}{A + jBy_{-1}}. \quad (34)$$

The simplified form for y_0 comes about because $y_0 = \infty$ for a circulator for which $s_0 = -1$.

Expanding the above equations in terms of $t = \tan \psi_0$, and putting

$$y_{\pm 1} = j(\lambda \mp \mu)$$

gives

$$t = -D/B \quad (35)$$

$$\frac{t - 3^{1/2}}{1 + t(3)^{1/2}} = \frac{C - \lambda D + \mu D}{A + \lambda B - \mu B} = \frac{t - (C - D\lambda)/\mu B}{1 + t[(A + B\lambda)/\mu D]} \quad (36)$$

and

$$\frac{t + 3^{1/2}}{1 - t(3)^{1/2}} = \frac{C - \lambda D - \mu D}{A + \lambda B + \mu B} = \frac{t + (C - \lambda D)/\mu B}{1 - t[(A + \lambda B)/\mu D]}. \quad (37)$$

These equations are consistent provided

$$y_1 \cot \Theta = \frac{CD - AB}{B^2 + D^2} \quad (38)$$

and

$$(3)^{1/2} y_{+1}' \tan \Theta_{+1} = \frac{AD + BC}{B^2 + D^2} \quad (39)$$

which must be evaluated at $2^{1/2} \cos \Theta = \cos \Theta_0$. These are the equations previously derived in [8].

To obtain y_t it is necessary to directly evaluate Γ_{11} at the center frequency in terms of the original variables. For $g < y_t^2$ the result is

$$(3)^{1/2} y_{+1}' \tan \Theta_{+1} = y_t^2[(2 - r)/r]^{1/2}. \quad (40)$$

In terms of the original variables (38)–(40) become

$$g = \frac{[r/(2 - r)]^{1/2} - \sin^2 \Theta}{[r/(2 - r)]^{1/2} \cos^2 \Theta} \quad (41)$$

$$y_1 = \{g[(2 - r)/r]^{1/2}\}^{1/2}\{g[r/(2 - r)]^{1/2} - 1\} \sin^2 \Theta \quad (42)$$

$$y_t^2 = [r/(2 - r)]^{1/2} g. \quad (43)$$

The present results are identical with those previously derived in [8] provided

$$r \approx [r/(2 - r)]^{1/2} \quad (44)$$

which is a good approximation for the values of r usually encountered in circulator design.

QUARTER-WAVE COUPLED-BELOW-RESONANCE STRIPLINE CIRCULATOR

The theory developed so far will now be combined with the electromagnetic problem in the case of the stripline circulator [1], [3], [11], [13], [14]

$$y_{+1} = \frac{-j\pi Y_e}{3Y_0 \sin \psi} \cdot \frac{\{J_1'(kR) - (K/\mu)[J_1(kR)/kR]\}}{J_1(kR)} \quad (45)$$

$$y_{-1} = \frac{-j\pi Y_e}{3Y_0 \sin \psi} \cdot \frac{\{J_1'(kR) + (K/\mu)[J_1(kR)/kR]\}}{J_1(kR)} \quad (46)$$

$$y_0 = \frac{j\pi Y_e}{3\psi Y_0} \left[\frac{J_0'(kR)}{J_0(kR)}\right]. \quad (47)$$

Fig. 6 depicts the preceding eigenadmittances as a function of kR for a circulator which when magnetized gives $r = 1.07$. The phase angles of the eigenreflection coefficients are shown in Fig. 7. These results show that the equivalent circuits for y_{+1} and y_{-1} are short-circuited radial

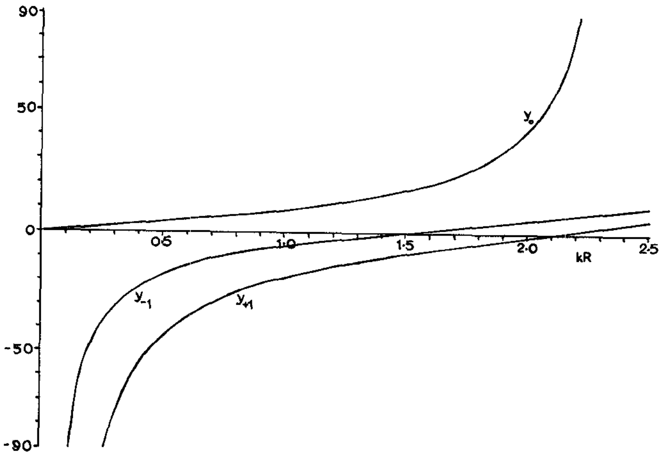

Fig. 6. Eigenadmittances of stripline circulator with $r = 1.07$, $2\delta_0 = 20$ percent.

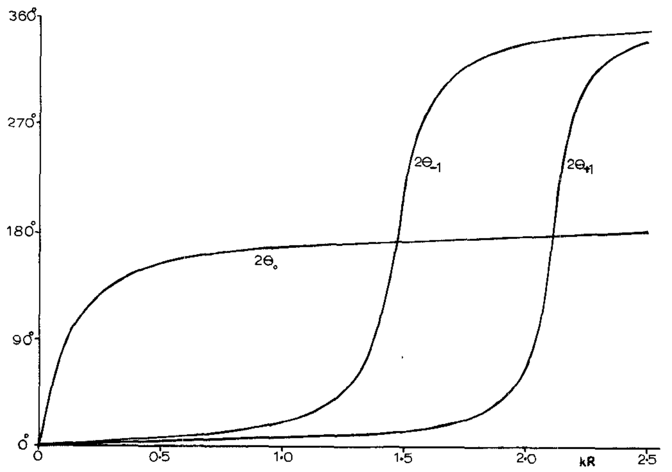

Fig. 7. Electrical lengths of eigennetworks for stripline circulator with $r = 1.07$, $2\delta_0 = 20$ percent.

transmission lines, while that of the eigennetwork y_0 is an open-circuited transmission line. If it assumed from the electromagnetic problem, in the usual way, that the frequency variation of the y_0 network can be omitted compared to the $y_{\mp 1}$ networks as was done in the network problem, the two are related.

The result is

$$g = (3)^{1/2} y_{+1}' \tan \Theta_{+1} = \frac{\pi Y_e}{(3)^{1/2} Y_0 kR \sin \psi} \cdot \frac{K}{\mu} \quad (48)$$

$$b' = \frac{\pi y_1}{4} = \frac{\pi Y_e}{3 Y_0 \sin \psi} \left[\frac{(kR)^2 - 1}{2(kR)} \right] \quad (49)$$

where it has been assumed that the susceptance slope parameters for the two equivalent reciprocal circuits are the same. Here g is the normalized gyrator admittance of the junction, and b' is the normalized susceptance slope parameter of the reciprocal eigennetworks.

The electromagnetic problem that must be satisfied when the frequency variation of the s_0 eigenvalue is neglected compared to that of $s_{\pm 1}$ is therefore, in the case of a quarter-wave coupled circulator,

$$\frac{\pi Y_e}{3 Y_0 \sin \psi} \cdot \left[\frac{(kR)^2 - 1)}{2kR} \right] = \frac{\pi}{4} \left[\frac{r - \sin^2 \Theta_0}{r^2 \cos^2 \Theta_0} \right] (r - 1) \tan^2 \Theta_0 \quad (50)$$

$$\frac{\pi Y_e}{(3)^{1/2} Y_0 kR \sin \psi} \cdot \frac{K}{\mu} = \frac{r - \sin^2 \Theta_0}{r \cos^2 \Theta_0}. \quad (51)$$

The first equation determines $\sin \psi$ and the second gives K/μ.

In the preceding equations [13]–[15]

$$Y_e = 4(\epsilon_r \epsilon_0 / \mu_e \mu_0)^{1/2} \left[\ln \left(\frac{W + b}{W + t} \right) \right]^{-1} \quad (52)$$

$$Y_0 = 4(\epsilon_0 / \mu_0)^{1/2} \left[\ln \left(\frac{W + b}{W + t} \right) \right]^{-1} \quad (53)$$

$$\sin \psi = \frac{W}{2R} \quad (54)$$

where W is the width of the stripline, b is the ground-plane spacing, t is the thickness of the center conductor, and the other quantities have the usual meanings.

The width of the center conductor is now obtained from (54), and for a 50-Ω line the ground-plane spacing b and center conductor thickness are obtained from (53) [15].

FREQUENCY VARIATION OF QUARTER-WAVE COUPLED CIRCULATOR

The assumption used throughout this paper is that the frequency variation of the s_0 eigenvalue may be omitted compared to that of the $s_{\pm 1}$ eigenvalues. This assumption will now be tested in the case of stripline circulators with $2\delta_0 = 20$ percent, and $r = 1.10$ and 1.22. The physical variables $\sin \psi$ and K/μ in (45)–(47) are given by (50) and (51) once the bandwidth and VSWR are stated.

Figs. 8 and 9 give the frequency behavior of the overall quarter-wave coupled circulator for the ideal and actual cases. These results have been obtained by assuming that the center frequency lies midway between the two split frequencies rather than at $kR = 1.84$.

The above results show that although the frequency variation of the in-phase reflection coefficient is small compared to that of the counterrotating ones, it cannot be ignored for devices with relatively wide bands and low VSWR's.

FREQUENCY RESPONSE OF QUARTER-WAVE COUPLED CIRCULATOR WITH CAPACITIVE TUNING

One way in which the theoretical results can be obtained in practice is to introduce a thin metal post in the center of the junction. Equivalently, thin metal posts may be introduced at the input transformer terminals which is the normal approach in stripline devices. This last statement comes about because the in-phase eigennetwork is an open-circuited half-wave resonator which can be tuned at either input or output terminals.

The $ABCD$ matrix for a single shunt capacitor is

$$A = 1 \quad (55)$$

$$B = 0 \quad (56)$$

$$C = \frac{\omega C}{Y_0} \quad (57)$$

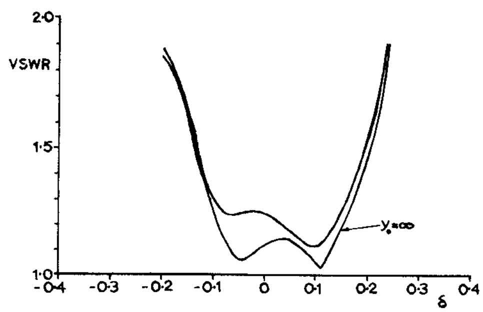

Fig. 8. Frequency response of quarter-wave coupled circulator with and without third eigennetwork for $h = 1.10$, $2\delta_0 = 20$ percent.

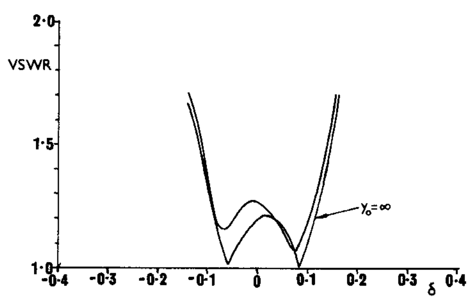

Fig. 9. Frequency response of quarter-wave coupled circulator with and without third eigennetwork for $r = 1.22$, $2\delta_0 = 20$ percent.

$$D = 1. \tag{58}$$

Cascading this $ABCD$ network with that for the single quarter-wave transformer gives the following $ABCD$ parameters for the overall matching network:

$$A = \cos \Theta \tag{59}$$

$$B = \frac{\sin \Theta}{y_t} \tag{60}$$

$$C = y_t \sin \Theta + \frac{\omega C}{Y_0} \cos \Theta \tag{61}$$

$$D = \cos \Theta - \frac{\omega C}{Y_0} \frac{\sin \Theta}{y_t}. \tag{62}$$

The arrangement used here is shown in Fig. 10. Figs. 11 and 12 indicate the influence of the capacitor C on the overall frequency response of the device for $r = 1.10$, 1.22, and $2\delta_0 = 20$ percent. They show that such a capacitor can indeed be used to improve the correlation between the two- and three-eigennetwork models of the stripline circulator.

FREQUENCY RESPONSE OF A RECIPROCAL 3-PORT JUNCTION

It is also possible to obtain the frequency response of the reciprocal junction by taking a linear combination of the eigenreflection coefficients of the junction [12]. The result is

$$3\Gamma_{11} = \gamma_0 + \gamma_{+1} + \gamma_{-1}$$

$$= \gamma_0 + 2\gamma_1 \tag{63}$$

where

$$\gamma_0 = \exp(-j2\psi_0) \tag{64}$$

$$\gamma_1 = \exp[-j2(\psi_1 + \pi/2)] \tag{65}$$

and

$$j \tan(\pi/2 + \psi_0) = \frac{jC + Dy_0}{A + jBy_0} \tag{66}$$

$$j \tan(\pi/2 + \psi_1) = \frac{jC + Dy_1}{A + jBy_1}. \tag{67}$$

CONCLUSIONS

This paper has developed a scattering theory for 3-port junction circulators with Chebyshev characteristics. This theory allows the influence of the in-phase eigenvalue of the 3-port circulator on the overall response of the device to be studied. The main conclusion of the paper is that while the in-phase eigenvalue of the junction may be omitted for devices with moderate specifications, it cannot be neglected for high-quality ones. Indeed, it may be necessary to introduce an additional independent variable in the form of a variable capacitor at the input terminals

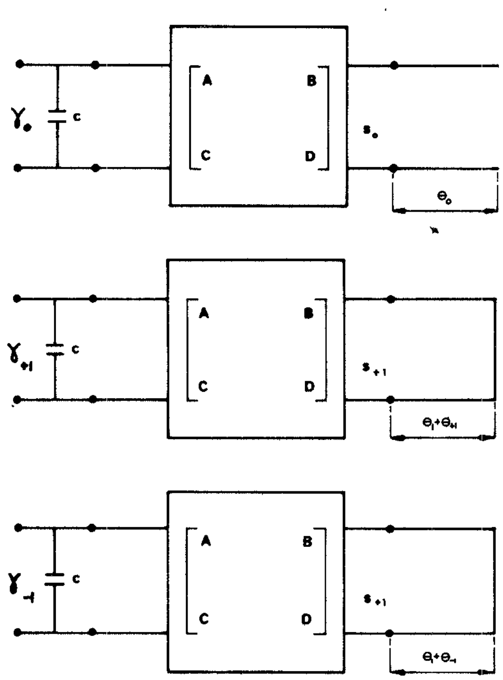

Fig. 10. Eigennetworks of 3-port circulator with capacitive tuning.

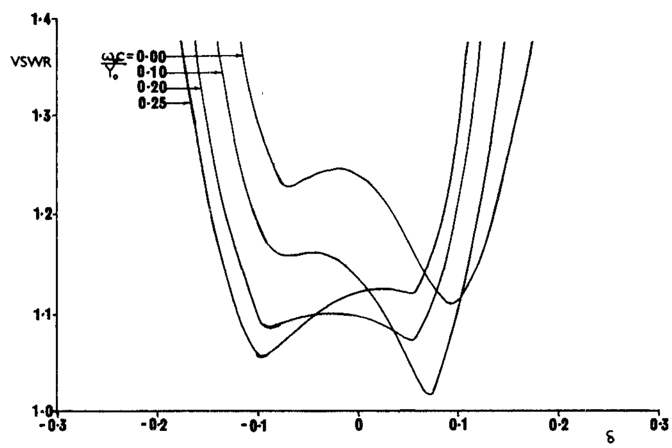

Fig. 11. Frequency response of quarter-wave coupled circulator with capacitive tuning for $r = 1.1$, $2\delta_0 = 20$ percent.

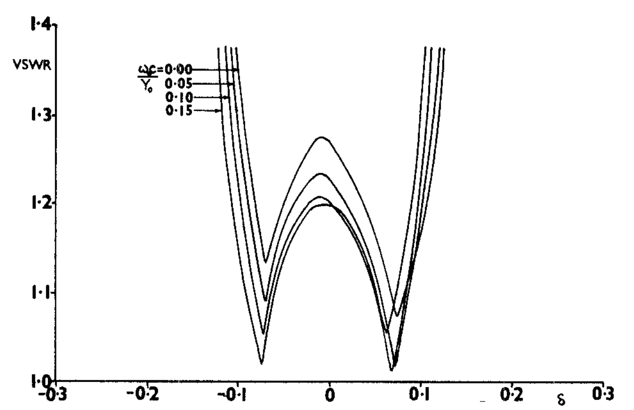

Fig. 12. Frequency response of quarter-wave coupled circulator with capacitive tuning for $r = 1.22$, $2\delta_0 = 20$ percent.

of the junction to optimize its electrical length. The approach described in this paper may also be used to investigate the influence of tolerance on the overall performance of stripline circulators.

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